Taxicab Travel: Mathematically Touring Baltimore

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In this article, the authors explore how to engage students in critical thinking through a virtual field trip. From the mathematics classroom, students problem solve using surface area of three-dimensional shapes, non-Euclidean geometry, probability, and Euclidean constructions. By the end of these activities, students have applied their mathematics understanding to mummies, travel, code breaking, and treasure hunting.

Introduction

"I'm dead. Sorry, I wasn't thinking!" is not the traditional statement a teacher hears in a high school classroom. Instead, two questions frequently heard are: "When are we ever going to use this?" and "Can we go on a field trip?" Unfortunately for the subject of mathematics, field trips can be difficult. Math is the building block of the world surrounding us, but it can be a challenge to create an interesting field trip relevant to the subject. However, it is the ubiquity of math that allows teachers to take their classes on a field trip without ever leaving the classroom. A virtual field trip allows students to have fun while teachers effectively present the material they want their students to learn. While our activities are based in Baltimore, any city can serve as the setting for a mathematical field trip. Our lesson is easily adaptable for incorporating different geometric lessons, as can be seen in each activity sheet.

Wrapping Mummies

Our virtual field trip followed a storyline to connect the activities. We began at the Walters Art Gallery where the mummies on display inspired the students to learn the principles of wrapping mummies. The students were given the task of wrapping a mummy when Egypt had a budget crisis. To make sure no materials were wasted, students calculated how much material they needed before wrapping the mummy. The students worked in groups of three or four, using traditional formulas to approximate the surface area of a doll. Groups first decomposed the doll into basic, three-dimensional shapes and then found their surface area using string and a ruler to take measurements. For this activity, students wrapped each limb individually as shown in Fig 1.

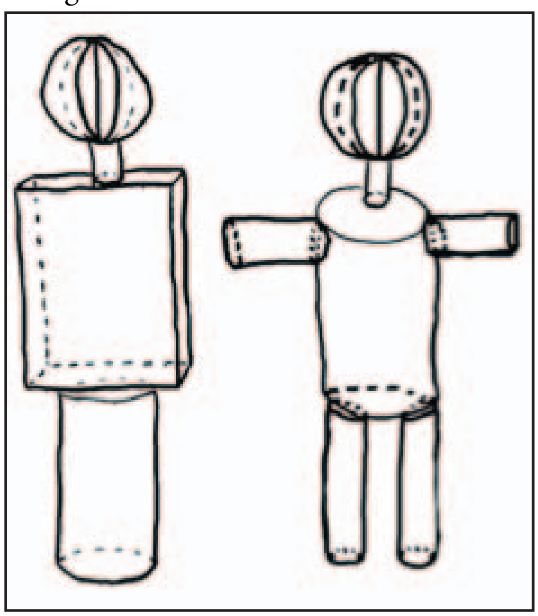


Fig 1 The decomposition of a human (left) and doll (right) when wrapping

We explained that Egyptian mummies first had their arms and legs wrapped separately. Next, the legs were wrapped

Groups first decomposed the doll into basic, three-dimensional shapes and then found the surface area of these shapes using string and a ruler to take measurements of the doll.

together and the arms were wrapped with the torso, and students subtracted any overlapping areas. For example, the neck is a cylinder but when wrapping, the bases of the neck are not wrapped, so their areas must be subtracted. Determining what areas to subtract from each solid was a challenge for many students. Some members automatically excluded the top and bottom of the cylinder for the neck, while others found the surface area of the whole cylinder, planning to subtract the area of the bases at the end. Because the cylinder of the neck also meets the sphere, which is the head, the students estimated the portion of the sphere covered by the neck (and thus not included in the total surface area). This led to discussions about the "best way" to calculate the total area, and provided an opportunity for group members to communicate and discover where they were inconsistent. Then, the dolls were wrapped in painter's tape, with as little overlap as possible, in order to approximate the actual surface area by measuring the amount of tape used. Students used varied strategies for this process. Some groups wrapped the doll and then measured the tape they used after they finished. Others measured strips of tape before wrapping and kept track of how much they used.

Whether it was with a doll or a classmate, by decomposing the body into familiar solids, students satisfied the NCTM standards using problem solving, visualization, spatial reasoning, and geometric modeling.

In some classes, we had groups not only wrap dolls in tape, but also wrap a group member in toilet tissue. Like the groups wrapping the dolls, these groups had similar discussions when trying to calculate their initial surface area predictions. When wrapping a student, the legs were wrapped together as a cylinder and the arms were wrapped with the torso as a rectangular prism [See Fig 1]. This process was more complicated (often working best with two students wrapping one mummy), forcing students to communicate in order to minimize overlap in the toilet tissue. Students had more fun with this method; as one participant put it, "It helps if the person is dead!" After groups finished wrapping one of their members, we observed numerous ways in which these groups found the area of the toilet tissue they used. While some groups used a more "traditional" method that involved finding the area of one square of the tissue and then multiplying that area by the total number of squares used, other groups used floor space to help them calculate the total area. One group tore their total length of toilet tissue into strips of equal length to create a large rectangle. Other groups, realizing that the floor tiles were each a foot long, stretched their one continuous strip of tissue out in the hallway, counting the tiles it covered to measure its length. Whether it was with a doll or a classmate, by decomposing the body into familiar solids, students satisfied the NCTM standards of using problem solving, visualization, spatial reasoning, and geometric modeling. In order to do this, they must know the surface area formulas for spheres, cylinders, and prisms as required in the NCTM Measurement standard that states students will apply appropriate techniques, tools, and formulas to determine measurements.

Looking for a Graveyard

Within the scope of this virtual field trip, the students overheard the curator of the Walters Art Museum speaking about a treasure. The students were told it was hidden somewhere in Baltimore and that their goal was to find its location. They came across a set of clues and a map of a portion of the city. Using the clues (see *Taxicab Geometry Student Worksheet*), which provided directions in Taxicab Geometry, the students worked to find their way to the next location.

Taxicab Geometry was developed as the geometry used by taxi drivers moving

Taxicab Geometry Student Worksheet Name: ____________________

Directions: Complete the items below to determine the location of the Edgar Allen Poe Museum and Graveyard.

Item 1

CLUE: The Cone Collection at the Baltimore Museum of Art features 500 works by Henri Matisse. Divide this number by 125. DIRECTION: Move this many Taxicab units from your current location (the intersection of Centre and Charles Streets)—half to the south, half to the west. This will leave you more than two Euclidean units from your original point.

Item 2

CLUE: Find all points 5 units away (in Taxicab Geometry) from your current location, and *mark each of them on the grid below.* DIRECTION: Find the intersection of Baltimore and Charles Streets. Move to the southern-most point on the "circle", 3 units (in Taxicab Geometry) away from this intersection.

Item 3

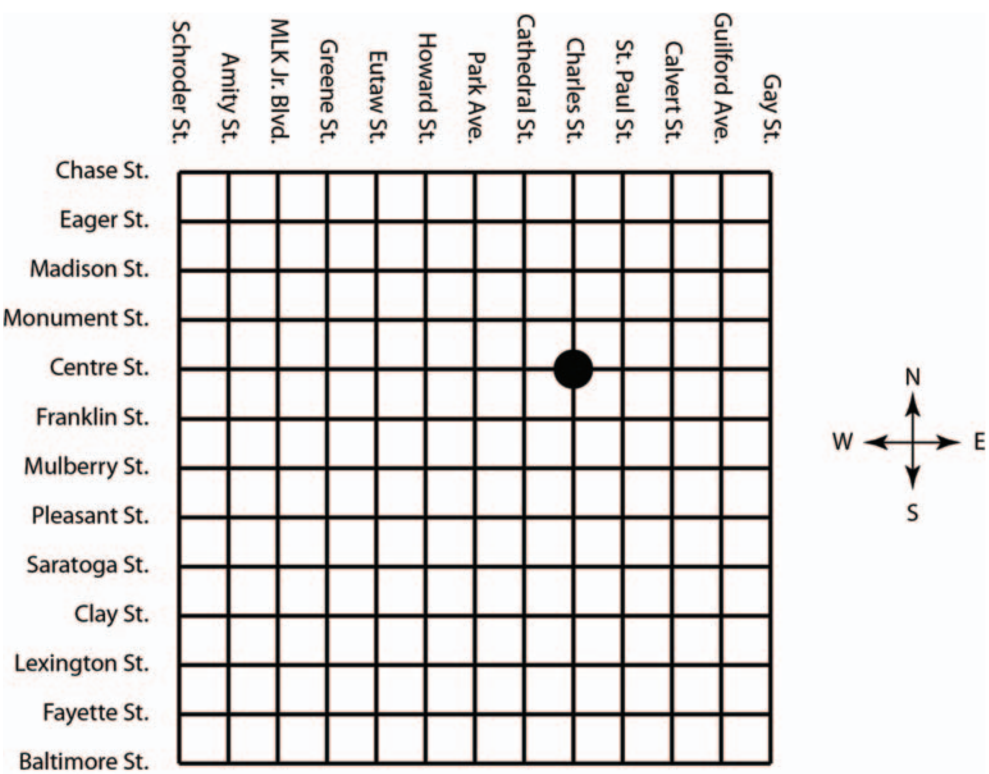
CLUE: Alfred Jensen is an artist featured at the Baltimore Museum of Art who uses numbers in his paintings. One of his paintings, "Coordinative Thinking on the Square and Rectangle," features a grid of numbers. One number is featured twice. These numbers are located side-by-side. DIRECTION: Using this duplicate number, move the number in the one's place units west, and the number in the ten's place units north.

Item 4

CLUE: At the Walters Art Gallery, we learned it takes about 40 days to completely wrap a mummy. Divide this number by 10. DIRECTION: Move this many units to the south. Then, go to the point from the Taxicab circle that is 2 units away from this location and is also the closest point to this location in Euclidean geometry.

Item 5

CLUE: From your current location, you see there is an accident between Eutaw and Greene Streets, which you have to avoid or you will be late! DIRECTION: To avoid this accident, move a total of 4 units in the southwesterly direction. You must do this by changing direction exactly once during travel, and moving more units to the west than to the south.



on the grid of city streets. On paper, it is illustrated on a grid consisting of a series of perpendicular horizontal and vertical lines. Movement occurs on lines, but not in between, just as we drive on roads but not through buildings. Because of this, the shortest distance between two points takes multiple paths. A circle is defined as the set of points a fixed distance from the center. In Euclidean Geometry this is a round shape we easily identify as a circle. Applying this definition to Taxicab Geometry, the set of points a fixed distance from the center creates a square. Initially, this concept was difficult for students, but the comparison to driving often made it easier for them to understand. When driving, we rarely make a Euclidean circle. Instead, we continue to make left or right turns until we end up in the same location; in other words, we drive in a square. High school students are close to driving age (if not of driving age), and connecting geometry to this major event in their lives piques their interest and understanding.

> The decryption is done using frequency analysis, a method that uses probability and logic to decipher a text based on the location and number of occurrences of each enciphered character.

In this activity, students plotted points in Taxicab Geometry on a grid (see *Taxicab Geometry Student Worksheet*), fulfilling the NCTM Geometry standard to specify locations and describe spatial relationships using coordinate geometry and other representational systems. As a non-Euclidean geometry, many students were unfamiliar with Taxicab Geometry prior to this lesson. The introduction of this geometry system enabled students to see mathematics beyond what is normally covered in the high school curriculum. This activity was designed for students to draw their own comparisons between Euclidean and Taxicab Geometry, such as how a circle is represented in Taxicab Geometry. We found placing this comparison toward the end of the activity was effective because students understood the properties of Taxicab geometry better once they had used it to solve several clues. This activity forced students to see Euclidian concepts such as circles and the shortest distance between points in a manner that required them to understand the concepts' true definitions, not just their appearances or properties. The activity concluded with the students reaching the Edgar Allen Poe Museum and Graveyard.

Code Breakers

Once the students reached the graveyard, they were greeted with an encrypted set of clues (see *Code Breaking Student Worksheet*). Their task was to decrypt the clues and follow them to find the treasure. The decryption is completed using frequency analysis, a method that uses probability and logic to decipher a text based on the location and number of occurrences of each enciphered character. For example, because "e" is the most common letter used in the English language, the letter that appears most frequently in the encryption most likely represents "e." Also, because "a" and "I" are the only one-letter English words, these letters can also be easily identified. We gave students a chart that had the occurrences of each encrypted letter already counted along with some of the less commonly used letters as shown in the *Code Breaking Student Worksheet*. This required students to apply their knowledge of the most frequent letters and words in the English language, which we discussed as a class, to decipher the clues. By analyzing the frequency of letters, this activity fulfills the NCTM Data Analysis & Probability standard to understand and apply basic concepts of probability. In this activity, we observed several methods by which students chose to decrypt the text. Some students decrypted the clues one letter at a time while others decrypted entire words and then looked for every occurrence of the word to decipher. Once the decryption

Code Breaking Student Worksheet   Name: ____________________

Directions: Use frequency analysis to decypher each clue. Then use the clues to find the hidden treasure on the map on the following page.

Clue 1: FA MARPW MU RGHWR IG IXA LGOJIX RJPKA HTTAYHPIANU IG GOJ WGJIX, PWY FAWI IG IXA WGJIXAPBI DGJWAJ GL IXA RJPKA.

Clue 2: FA IXAW IJPKANNAY LGJIU-LHKA YARJAAB BGOIXAPBI LGJ PW ABIHTPIAY IF-AWIU-AHRXI LAAI, PWY TGKAY APBI PNGWR IXA BHYA GL IXA RJPKA OWIHN FA JAP-DXAY HIB WGJIXAPBI DGJWAJ.

Clue 3: LJGT XAJA, FA FAWI IG IXA TGBI APBIAJW SGHWI GL IXA IJAA IG IXA WGJIX GL GOJ DOJJAWI SGBHIHGW, PWY FA TAPBOJAY IXA YHBIPWDA FA IJPKANNAY.

Clue 4: FA DGWWADIAY GOJ DOJJAWI SGBHIHGW PI IXA IJAA IG IXA DGJWAJ GL IXA RJPKA LJGT FXHDX FA NALI, PWY IXAW OBAY IXPI DGJWAJ GL IXA RJPKA PB IXA THYSGHWI GL P NHWA BARTAWI.

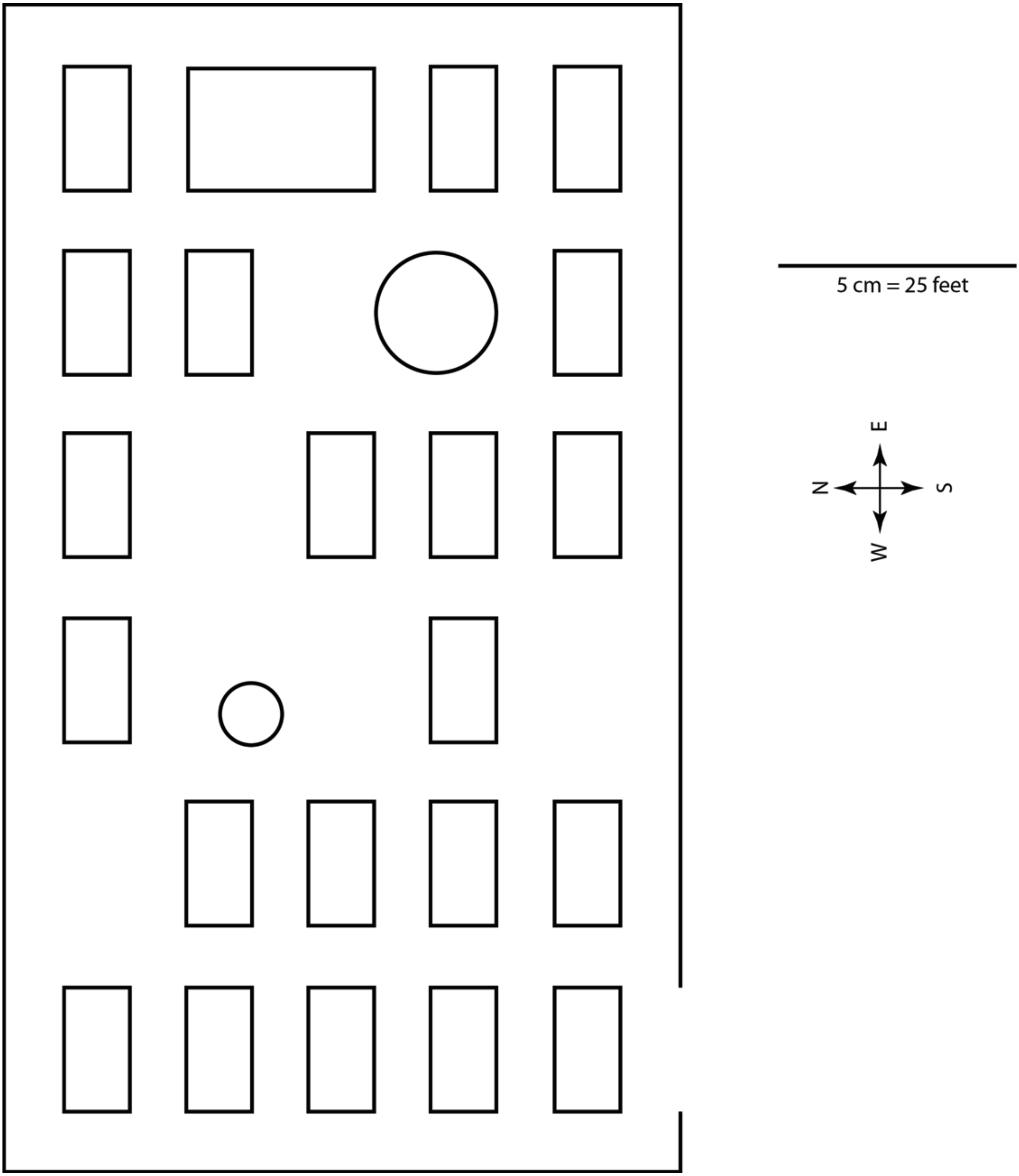
Clue 5: FA DGWBIJODIAY IXA SAJSAWYHDONPJ MHBADIGJ GL IXA AWIHJA NHWA BAR-TAWI IXPI FA DGWBIJODIAY, PWY FPNVAY APBI PNGWR IXHB SAJSAWYHDONPJ LGJ PSS-JGCHTPIANU IXHJIU-BAKAW LAAI.

Clue 6: LHWPNNU, FA IOJWAY WHWAIU YARJAAB IG IXA WGJIX PWY LGNNGFAY IXHB SPIX IG IXA RJPKA FXAJA IXA IJAPBOJA FPB XHYYAW.

Encrypted	Occurences	Original	Encrypted	Occurences	Original	Encrypted	Occurences	Original
A	119		J	56	R	S	11	
B	29	S	K	11	V	T	12	M
C	1	X	L	20	F	U	8	Y
D	18	C	M	3	B	V	1	K
E	0	Z	N	19		W	51	
F	19	W	O	16	U	X	41	
G	53	O	P	43		Y	29	
H	31		Q	0	J	Z	0	Q
I	81		R	16				

Code Breaking Student Worksheet Name: ____________________

Directions: Use your deciphered clues to find the hidden treasure on the map provided below.



was complete, the students had clues that utilized Euclidean constructions (including perpendicular bisectors, angles, and line segments) to find the grave where the treasure was hidden on the map provided in the *Code Breaking Student Worksheet*. After finding the exact location of the treasure, one student exclaimed, “This math stuff really works!” which is a result the field trip is designed to elicit. While using compasses, protractors, and rulers to find the treasure, students fulfill the NCTM standard to understand measurable attributes of objects and the units, systems, and processes of measurement.

In Summary

Numerous indicators in the Problem Solving, Communications, and Connections standards were met in each activity of our virtual field trip. As the students were presented with real life situations, they worked together to find creative solutions to each problem in order to reach their goal. The students judged the reasonableness of numerical computations and their results. As a result, they learned to conduct error analysis from real world situations.

As four undergraduate students at Towson University, we created this lesson as a capstone project for our Honors College experience. We were given the opportunity to test these activities with numerous classes in high schools in Baltimore and Frederick Counties in Maryland. We also presented this lesson to a class of college students taking an honors seminar in mathematics at Towson University. We refined our activities after working with each class in order to present our lesson at a professional development workshop for high school teachers in Baltimore County and at the NCTM regional conference in Baltimore, MD, in October 2010. Our participants for this lesson ranged from high school students in advanced placement math classes to undergraduate students not majoring in math, from teachers with only a few years of experience to assistant deans of universities. Regardless of the participant’s abilities, the same problems had to be solved. Whether it was approximating the surface area of the mummy before wrapping or discovering what a circle is in Taxicab Geometry, all of our learners were engaged and thought critically throughout our activities.

The benefits students obtained from these activities are plentiful. Aside from the principles of geometry and problem solving utilized, they were able to enjoy mathematics in an entertaining classroom activity that allowed their minds to travel outside of the box. This showed students that math is all around them and that they can use it to help them in some unlikely situations. Our ultimate goal from this lesson was for students to find math in other aspects of their lives, whether it is when they are walking through a museum or just figuring out the best way to get from one place to another. With the idea of the virtual field trip, we are left with only one question: Where will you take your students?

One student exclaimed, “This math stuff really works!” which is the exact result the field trip is designed to elicit.

Decoded messages from Code Breaking Student Worksheet

Clue 1: We began by going to the fourth grave immediately to our north, and went to the northeast corner of the grave; **Clue 2:** we then travelled forty-five degrees southeast for an estimated twenty-eight feet, and moved east along the side of the grave until we reached its northeast corner; **Clue 3:** from here, we went to the most eastern point of the tree to the north of our current position, and we measured the distance we travelled; **Clue 4:** we connected our current position at the tree to the corner of the grave from which we left, and then used that corner of the grave as the midpoint of a line segment; **Clue 5:** we constructed the perpendicular bisector of the entire line segment that we constructed, and walked east along this perpendicular for approximately thirty-seven feet; **Clue 6:** finally, we turned ninety degrees to the north and followed this path to the grave where the treasure was hidden.

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